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A New Multi-Criteria Decision Making Based on Fuzzy- Topsis Theory

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Abstract — **In this paper, a new extended method of multi criteria decision making based on fuzzy-Topsis theory is introduced. Mostly, it is not possible to gather precise data, so decision making based on these data loses its efficiency. The fuzzy theory has been used to overcome this draw back. In multi-criteria decision making, criteria can correlate with each other, most of which are ignored in classic MCDM. In this paper, correlation coefficient of fuzzy criteria has been studied to adapt the interrelation between criteria and a new algorithm is proposed to obtain decision making. Finally the efficiency of suggested method is demonstrated with an example.**

Key words - **MCDM, correlation, fuzzy-Topsis.**

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I. INTRODUCTION

Decision making is the process of selecting the most appropriate choice among many others. One of the main branches of decision making science is multi criteria decision making (MCDM). In MCDM more than one criterion is important for the best choice. These criteria can be qualitative, quantitative, and positive or negative [1-4]. When it's hard or impossible to get precise data, fuzzy theory can be used as an appropriate and strong tool for analyzing ambiguous and imprecise problems [5]. One of the most common ways of MCDM is Topsis. Topsis is clear and understandable with no complexity. In Topsis, criteria weights and choice efficiencies should be precise but in practice it's not so. Therefore, most of the researchers try to apply fuzzy data in Topsis. In most fuzzy Topsis methods, some fuzzy data has been definitely eliminated, so some information has been lost as well. Izadikhan [6] has developed Topsis method for decision making in one interval or fuzzy data. Mahdavi et al. [7] has proposed fuzzy- Topsis with transforming fuzzy data to non-fuzzy data. Wang and Elhag [8], Ding and Chou [9] have generalized fuzzy-Topsis on the basis of $\alpha - cut$. Chen has solved fuzzy-Topsis method in one interval one leniency reduction in [10]. Yue [11] extended Topsis with interval numbers.

Three important phases in all MCDM methods for ranking are as follow:

1. Determining criteria and different choices.
2. Attributing Prices to weights of criteria and determining choices rate in proportion to different criteria.
3. Processing numerical Prices for determining

ranks of choices.

Most researchers emphasize on the second and third stages but the first one has not attracted much attention. Sometimes surveying to choose an appropriate criterion is ignored and it becomes optional. Some researchers select inter related criteria and so it leads to numerous criteria. Moreover, it becomes boring and hard to analyze the criteria because of repetitive evaluations and a lot of comparisons. So, correlation coefficient between variables is also studied to remove variables with high inter relation amounts. Chaudhur and Bhattachary [12] have used Spearman correlation coefficient for calculating correlation of two fuzzy series. Hung and Wu [13] have applied Centroid method to calculate correlation of two fuzzy series. They have shown that, these relations can be positive or negative. Urdakul and Tarselic [14] have used Spearman coefficient for crisp numbers. In this paper a new method of fuzzy-Topsis is proposed where fuzzy numbers are ranked directly. It also has been used to find the correlation between variables and to remove highly related values. Next sections are as follows: section 2 includes primary definition of subject. Section3 describes correlation coefficients between criteria and section 4 includes suggested algorithm. The efficiency of suggested method is demonstrated in section 5 by means of an experimental example.

II. PRIMARY DEFINITIONS

Topsis method has been developed by Wang and Lee [14]. Its rule is such that the selected choice has the least distance of positive ideal solution and the farthest from negative ideal solution [1, 2]. Multi criteria decision making (MCDM) methods have the benefit of evaluating different choices. They can also analyze and evaluate qualitative and quantitative criteria at the same time. A MCDM problem can be summarized in a decision matrix as shown in Fig.1.

	C_1	C_2	…	C_n
A_1	$\tilde{x}_{11}$	$\tilde{x}_{12}$	…	$\tilde{x}_{1n}$
A_2	$\tilde{x}_{21}$	$\tilde{x}_{22}$	…	$\tilde{x}_{2n}$
…	…	…	…	…
A_m	$\tilde{x}_{m1}$	$\tilde{x}_{m2}$	…	$\tilde{x}_{mn}$

$$\widetilde{W} = [\widetilde{w}_1, \widetilde{w}_2, \dots \widetilde{w}_n]$$

Fig.1 Decision matrix

Where alternatives $(A_1, A_2 \dots A_m)$ are options and $C_1, C_2 \dots C_m$ are decisions making Criteria. Utility of each choice regarding the criterion is referred by X_{ij} and W_j is the weight (importance factor) of criterion c_j. The main purpose of decision making mechanism is selecting the best choice from alternatives, A_i , such that the selected alternative has the highest rank and efficiency. The introduced method is fuzzy stated with fuzzy numerical choices.

Definition1. (Triangular fuzzy number): fuzzy number A is referred as $\tilde{A} = (a, b, c)$ of crisp numbers with (a<b<c).The membership function of triangular fuzzy number is defined as below:

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{x-a}{b-a} & if \quad a \le x \le b \\ \dfrac{x-c}{b-c} & if \quad b \le x \le c \\ 0 & otherwise \end{cases} \qquad (1)$$

Definition2. A fuzzy number $\widetilde{A}$ is called a positive fuzzy number if $\mu_{\widetilde{A}}(x) = 0$ for all $x < 0$

Definition3. If $\widetilde{A}$ is a triangular fuzzy number and $[\widetilde{A}]^L_\alpha > 0$ *and* $[\widetilde{A}]^U_\alpha \le 1$ for $\alpha \in [0,1]$, then is called a normalized positive triangular fuzzy number.

Note1. If $\widetilde{A} = \left[[\widetilde{A}]^L_\alpha, [\widetilde{A}]^U_\alpha\right]$, then by choosing $\alpha = 1$ we can identify the center value of $\widetilde{A}$, and by $\alpha = 0$ we can identify the left and right extension of $\widetilde{A}$.

III. CORRELATION COEFFICIENT

Measuring correlation coefficient between two variables is important since it shows strength and rate of relationship between two variables. For example there is correlation between the student's math grade and statistics grade.

In this paper we'll use fuzzy numbers presented in [16] to calculate the correlation coefficients. In [16] credibility theory has been used to calculate correlation coefficient between two triangular numbers. Credibility theory is a branch of mathematics used to study behaviors of fuzzy numbers. If $M = (a_1, b_1, c_1)$ and $N = (a_2, b_2, c_2)$ are two triangular fuzzy variables, then correlation coefficient between N and M can be find by the following formula.

$$pc_r = \frac{(b_1 + a_1)(b_2 + a_2) + (c_1 + b_1)(c_2 + b_2)}{\sqrt{(b_1 + a_1)^2 + (c_1 + b_1)^2}\sqrt{(b_2 + a_2)^2 + (c_2 + b_2)^2}} \tag{2}$$

Where pc_r represents the correlation coefficient measures that have high correlation and are gained by the method presented in [13]. The pairs with correlation values more than .8 or less than -.8 are affiliated measures and are eliminate by following steps.

Calculate the correlation of a measure relating to all other measures as pairs.

List the criteria in columns and rows of correlation matrix.

3-The correlation coefficient is a measure to compare the pair with all other pairs of matrix and comparing one pair of the correlation coefficient with all other pairs. All other pairs that are correlated with current pair are omitted.

4- Repeat step 3 for all pairs of matrix.

SUGGESTED FUZZY ALGORITHM

The new algorithm for extending Topsis method in a fuzzy environment is as follows:

Step1: Determine evaluation criteria

Step2: Determine weight (attribute importance) of criteria

Step3: Determine decision alternatives

Step4: Calculate correlation coefficient between criteria's to remove dependent criteria using weights of criteria and the method proposed in [13].

Step5: Construct fuzzy decision matrix.

We assume that the fuzzy decision value for each $\tilde{x}_{ij}$ is a triangular fuzzy number. $\tilde{x}_{ij} = (a_{ij}, b_{ij}, c_{ij})$

Step6: Calculate the normalized fuzzy decision matrix.

At first, for each fuzzy number $\tilde{x}_{ij} = (a_{ij}, b_{ij}, c_{ij})$ we calculate the set of α-cut as: $\tilde{x}_{ij} = [[\tilde{x}_{ij}]^L_\alpha, [\tilde{x}_{ij}]^U_\alpha], \alpha \in [0,1]$

Therefore each frame is converted to a fuzzy number $\tilde{x}_{ij}$ by the proposed method in [9] which can be normalized as:

$$[\tilde{n}_{ij}]^l_\alpha = \frac{[\tilde{x}_{ij}]^l_\alpha}{\sqrt{\sum_{i=1}^{m}\left(([\tilde{x}_{ij}]^l_\alpha)^2 + ([\tilde{x}_{ij}]^u_\alpha)^2\right)}} \quad i = 1,\dots,m\ ,\ j = 1,\dots,n \tag{3}$$

$$[\tilde{n}_{ij}]^u_\alpha = \frac{[\tilde{x}_{ij}]^u_\alpha}{\sqrt{\sum_{i=1}^{m}\left(([\tilde{x}_{ij}]^l_\alpha)^2 + ([\tilde{x}_{ij}]^u_\alpha)^2\right)}} \quad i = 1,\dots,m\ ,\ j = 1,\dots,n \tag{4}$$

Now interval $[[\tilde{n}_{ij}]^L_\alpha, [\tilde{n}_{ij}]^U_\alpha]$ is a normal range of the interval $[[\tilde{n}_{ij}]^L_\alpha, [\tilde{n}_{ij}]^U_\alpha]$. According to note1 we can transform this normalized interval in to a fuzzy number such as $\tilde{N}_{ij} = (n_{ij}, a_{ij}, b_{ij})$, when $\alpha = 1$ we obtain $n_{ij} = [\tilde{n}_{ij}]^L_{\alpha=1} = [\tilde{n}_{ij}]^U_{\alpha=1}$, and when $\alpha = 0$ we have:

$$\begin{cases} [\tilde{n}_{ij}]^U_{\alpha=0} = n_{ij} + b_{ij} \\ [\tilde{n}_{ij}]^L_{\alpha 0} = n_{ij} - a_{ij} \end{cases}$$

Then:

$$\begin{cases} [\tilde{n}_{ij}]^U_{\alpha=0} = n_{ij} + b_{ij} \\ [\tilde{n}_{ij}]^L_{\alpha 0} = n_{ij} - a_{ij} \end{cases}$$

$\widetilde{N}_{ij}$ is a normalized positive triangular fuzzy number corresponding to $\tilde{x}_{ij}$.

Step7: Calculate nonsocial weighted matrix.

$$\widetilde{V} = \left(\widetilde{v}_{ij}\right)_{n*m}$$
$$\widetilde{V}_{ij} = \widetilde{N}_{ij}.\widetilde{W}_j \qquad i=1,\dots n \ , \ j=1,\dots,m \ ) \tag{5}$$

Where $\widetilde{W}_j$ is the weight of criterion C_j and $\sum_{j=1}^{m} w_j = 1$. [6, 17]

Step8: The largest triangular fuzzy number and the smallest one are calculated for each column of nonsocial weighted matrix. For finding the biggest and the smallest fuzzy number we apply the following relations proposed for trapezoidal numbers [10].

For each linear ranking function R we have $\widetilde{a} \geq_R \widetilde{b}$ if and only if $\widetilde{a} - \widetilde{b} \geq_R 0$ Also, if $\widetilde{a} \geq_R \widetilde{b}$ and $\widetilde{c} \geq_R \widetilde{d}$, then:

$$\widetilde{a} + \widetilde{c} \geq_R \widetilde{b} + \widetilde{d}$$

Therefore, we have the following linear ranking:

$$\widetilde{a} = (a^l, a^u, a, \beta)$$

$$R(\widetilde{a}) = c_l a^l + c_u a^u + c_\alpha.\alpha + c_\beta.\beta$$

Where $C_\beta, C_\alpha, C_u, C_L$ are constant numbers which at least one of them is nonzero Hence, if $\widetilde{b} = (b^l, b^u, \gamma, \theta)$ then, $\widetilde{a} \geq_R \widetilde{b}$ if and only if $a^L + a^u + \frac{1}{2}(\beta - \alpha) \geq b^L + b^u + \frac{1}{2}(\theta - \gamma)$

So for ranking Eq.s we have:

$$R(a) = a^l + a^u \frac{1}{2}(\beta - \alpha) \geq b^L + b^u + \frac{1}{2}(\theta - \gamma)$$

If we use triangular fuzzy number $\alpha = \beta$, then we have: [20]

$$R(a) = \frac{1}{2}(a^L + a^u) \geq \frac{1}{2}\left(b^l + b^u\right) \tag{6}$$

Step9: Calculate the ideal solution and negative ideal solution for each alternative.

$$A^+ = \left\{(\max_i \widetilde{V}_{ij} \mid j \in J) \mid i=1,2,\dots m\right\} = \left\{\widetilde{V}_1^+, \widetilde{V}_2^+, \dots, \widetilde{V}_j^+, \widetilde{V}_n^+\right\} \tag{7}$$

$$A^- = \left\{(\min_i \widetilde{V}_{ij} \mid j \in J) \mid i=1,2,\dots m\right\} = \left\{\widetilde{V}_1^-, \widetilde{V}_2^-, \dots, \widetilde{V}_j^-, \widetilde{V}_n^-\right\} \tag{8}$$

Step10: Find Euclidean distance of two triangular fuzzy numbers as [8, 9, 5]

The i-choice distance with ideals by using Euclidean method is:

$$\widetilde{d}_i^+ = (\widetilde{V}_{ij}, \widetilde{V}_j^+) = (\sum_{j=1}^{m} \left([\widetilde{V}_{ij}]_\alpha^L - [\widetilde{V}_i^+]_\alpha^L\right)^2 + \left([\widetilde{V}_{ij}]_\alpha^u - [\widetilde{V}^+{}_i]_\alpha^u\right)^2))^{\frac{1}{2}} \tag{9}$$

$$\widetilde{d}_i^- = d(\widetilde{V}_{ij}, \widetilde{V}_j^-) = (\sum_{j=1}^{m} \left([\widetilde{V}_{ij}]_\alpha^L - [\widetilde{V}^-{}_i]_\alpha^L\right)^2 + \left([\widetilde{V}_{ij}]_\alpha^u - [\widetilde{V}_i^-]_\alpha^u\right)^2))^{\frac{1}{2}} \tag{10}$$

Step11: Define proportional similarity of Aj with ideal solution as below:

$$\widetilde{R}_i = \frac{\widetilde{d}_i^-}{\widetilde{d}_i^+ + \widetilde{d}_i^-} \tag{11}$$

Step12: Select the decision choice with larger R_i. [6, 17]

We show the efficiency of suggested algorithm with an illustrative educational example.

Illustrative example:

University Professor's Ranking:

First, we defined fifteen criteria, then these criteria were analyzed by Payamenoor university students and the collected data were used to rank the three professors of the university.

Step1: Fifteen criteria are defined for evaluation as listed in Table 1.

TABLE 1. MEASUREMENT CRITERIA

C_1	*Does she/he have enough knowledge of the subject matter?*
C_2	*Does she/he use new scientific findings related to the subject matter?*
C_3	*Does she/he answer the students' questions?*
C_4	*Does she/he have the ability to state matters clearly?*
C_5	*Does she/he use existing facilities (board, text book, picture, chart, overhead…)?*
C_6	*Does she/he state clearly the objectives and subjects of the beginning of class and have cohesion or does she/he teaches as a lesson plan?*
C_7	*Does she/he perform in discussion with students or does she/he try to improve their intellectual productivity?*
C_8	*Does she/he manage the class well?*
C_9	*Does she/he use evaluation through educational term? Midterm exams, solving exercises, homework and project?*
C_{10}	*Does she/he encourage and motivate students to study and research?*
C_{11}	*Does she/he pay attention to student's logical suggestions and critics?*
C_{12}	*Is there availability to teacher at university to ask questions?*
C_{13}	*Does she/he obey cultural and reciprocal respect?*
C_{14}	*Is he/she on time and does she/he use the class time effectively?*
C_{15}	*Is she/he interested in teaching?*

TABLE 2. PROPORTIONAL IMPORTANCE OF CRITERIA

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	[0,0,...	[0,0.70...	[0,0,0.6...	[0.80...	[0,0,0.50...	[0.40...	[0.70...	[0,0,0.2...	[0.30...	[0.70...	[0,0,0.10...	[0.70...	[0,0...	[0,0,0.3...	[0,0,0.9...

TABLE 3. THE CORRELATION COEFFICIENTS FOR EACH PAIR OF CRITERIA

ciritera par	r	ciritera par	r	ciritera par	r	ciritera par	r
c_1-c_2	0.999969	c_3-c_4	0.997362	c_5-c_{10}	0.255893	c_8-c_{13}	1
c_1-c_3	0.999306	c_3-c_5	0.773957	c_5-c_{11}	1	c_8-c_{14}	1
c_1-c_4	0.999374	c_3-c_6	0.999695	c_5-c_{12}	0.901523	c_8-c_{15}	0.786318
c_1-c_5	0.749838	c_3-c_7	0.773957	c_5-c_{13}	1	c_9-c_{10}	0.255893
c_1-c_6	0.998083	c_3-c_8	0.773957	c_5-c_{14}	1	c_9-c_{11}	1
c_1-c_7	0.749838	c_3-c_9	0.773957	c_5-c_{15}	0.786318	c_9-c_{12}	0.901523
c_1-c_8	0.749838	c_3-c_{10}	0.810204	c_6-c_7	0.789352	c_9-c_{13}	1
c_1-c_9	0.749838	c_3-c_{11}	0.773957	c_6-c_8	0.789352	c_9-c_{14}	1
c_1-c_{10}	0.831471	c_3-c_{12}	0.971762	c_6-c_9	0.789352	c_9-c_{15}	0.786318
c_1-c_{11}	0.749838	c_3-c_{13}	0.773957	c_6-c_{10}	0.795489	c_{10}-c_{11}	0.255893
c_1-c_{12}	0.9623	c_3-c_{14}	0.773957	c_6-c_{11}	0.789352	c_{10}-c_{12}	0.649016
c_1-c_{13}	0.749838	c_3-c_{15}	0.999805	c_6-c_{12}	0.97729	c_{10}-c_{13}	0.255893
c_1-c_{14}	0.749838	c_4-c_5	0.725953	c_6-c_{13}	0.789352	c_{10}-c_{14}	0.255893
c_1-c_{15}	0.998375	c_4-c_6	0.995267	c_6-c_{14}	0.789352	c_{10}-c_{15}	0.798464
c_2-c_3	0.99898	c_4-c_7	0.725953	c_6-c_{15}	0.999988	c_{11}-c_{12}	0.9015
c_2-c_4	0.999623	c_4-c_8	0.725953	c_7-c_8	1	c_{11}-$c_1$4	1
c_2-c_5	0.744569	c_4-c_9	0.725953	c_7-c_9	1	c_{11}-c_{15}	1
c_2-c_6	0.997561	c_4-c_{10}	0.850612	c_7-c_{10}	0.255893	c_{11}-c_{16}	0.7863
c_2-c_7	0.744569	c_4-c_{11}	0.725953	c_7-c_{11}	1	c_{12}-c_{13}	0.9015
c_2-c_8	0.744569	c_4-c_{12}	0.952072	c_7-c_{12}	0.901523	c_{12}-c_{14}	0.9015
c_2-c_9	0.744569	c_4-c_{13}	0.725953	c_7-c_{13}	1	c_{12}-c_{15}	0.9762
c_2-c_{10}	0.835849	c_4-c_{14}	0.725953	c_7-c_{14}	1	c_{13}-c_{14}	1
c_2-c_{11}	0.744569	c_4-c_{15}	0.995733	c_7-c_{15}	0.786318	c_{13}-c_{15}	0.786
c_2-c_{12}	0.960114	c_5-c_6	0.789352	c_8-c_9	1	c_{14}-c_{15}	0.786
c_2-c_{13}	0.744569	c_5-c_7	1	c_8-c_{10}	0.255893		
c_2-c_{14}	0.744569	c_5-c_8	1	c_8-c_{11}	1		
c_2-c_{15}	0.997892	c_5-c_9	1	c_8-c_{12}	0.901523		

TABLE 4. THE CORRELATION BETWEEN THE TWO SETS FOR CRITERIA IS MARKED WITH *

	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}	C_{11}	C_{12}	C_{13}	C_{14}	C_{15}
C_1		*	*	*		*				*		*			*
C_2			*	*		*				*		*			*
C_3				*		*				*		*			*
C_4						*				*		*			*
C_5							*	*	*		*	*	*	*	*
C_6												*			
C_7								*	*		*	*	*	*	
C_8									*		*	*	*	*	
C_9											*	*	*	*	
C_{10}															
C_{11}													*	*	
C_{12}													*	*	*
C_{13}															
C_{14}														*	

TABLE 5. 3 SELECTED CRITERIA'S AFTER CALCULATING CORRELATION COEFFICIENT

	1	2	3
1	[0.7000,0.8000,0.9000]	[0,0,0.1000]	[0.7000,0.7000,0.7000]

TABLE 6. DECISION MATRIX FOR 3 CRITERIA

	1	2	3
1	[0,0,0.1000]	[0.4000,0.50...	[0,0,0.1000]
2	[0.2000,0.40...	[0,0,0.3000]	[0,0,0.3000]
3	[0.8000,0.85...	[0,0,0.7000]	[0.5000,0.50...

TABLE 7. NONSOCIAL WEIGHTED MATRIX AND THE LARGEST AND SMALLEST VALUE OF EACH COLUMN

	1	2	3
1	[0,0.0550,0....	[0,0,0.0953]	[0,0.0831,0....
2	[0.1027,0.55...	[0,0,0.0286]	[0,0.2492,0....
3	[0.4106,1.40...	[0,0,0.0667]	[0.4154,1.32...
max	[0.4106,1.40...	[0,0,0.0953]	[0.4154,1.32...
min	[0,0.0550,0....	[0,0,0.0286]	[0,0.0831,0....

Step2: There are three alternatives A1, A2, A3 (representing the first, second and third teachers) represented by number 1, 2 and 3 in Table

Step3: Define weights of criteria and 3 alternatives for decision. The results are shown in Table 2.

Step4: We compute coefficient correlation for every pair of criteria by Eq. (2). In Table 3, pairs of criteria and their coefficient correlations are shown.

The correlated criteria (with correlation coefficient more than 0.8) are shown in Table 4 marked by *.

For example criterion c1 is dependent to c2, c3, c4, c6, c10, c12 and c15. So, criteria c2, c3, c4, c6, c10, and c12 can be substituted by c1. This procedure repeats for other criteria and finally three criteria c5, c1, c13 remains for decision making which are shown in Table 5.

Step5 and step6: New fuzzy decision matrix is generated for the three remaining criteria as shown in Table 6.

Step7: Generate Nonsocial weight matrix weighed by using Eq. (5) as shown in Table 7

Step 8: Find the biggest and smallest triangular fuzzy numbers for each column according to Eq. (6) as shown in Table 7

Step 9: Find positive solution and negative solutions, according to Eq.s (7), (8).

Step10: Compute Euclidean distance of two triangular fuzzy numbers by using Eq.s (9) and (10). The results are shown in columns 1 and 2 in Table 8.

Step11 and step 12: Find the priority list of alternatives based on Ri (11), as shown in column 4 of Table 8, that is A1>A2>A3.

TABLE 8. RANKING DIFFERENT ALTERNATIVES

If we consider all criteria without calculating correlation coefficient and eliminate the correlated criteria results will be as listed in Tables 9, 10 and 11. Results of Tables 8 and 11 illustrate that the same ranking has been obtained for the criteria 3 and 15.

The ranking values of two methods are shown in Figure 2.

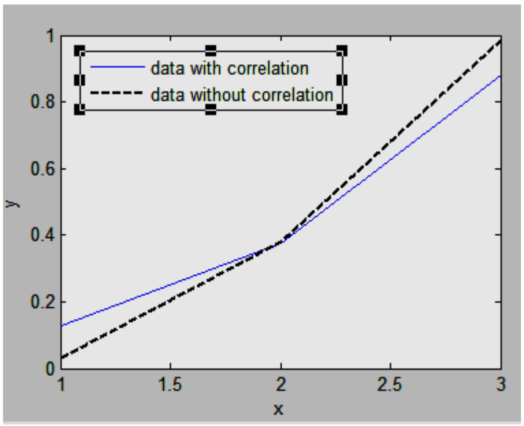

Fig. 2. The ranking values with and without the removal of correlated criteria

CONCLUSION

In this paper a new extension of Topsis is introduced for fuzzy multi criteria decision making (MCDM). MCDM is used as a solution for parallel programs possessing that have series of qualitative and quantitative estimates to rank different alternatives. This method covers both certain data and subjective judgments. The correlation coefficient between criteria is calculated to reduce the number of criteria. An experimental example is used to show the efficiency of introduces procedure.

Table 8. Ranking different alternatives

	1	2	3	4
1	2.0111	0.0667	0.0321	3
2	1.2908	0.7983	0.3821	2
3	0.0286	2.0115	0.9860	1

TABLE 9. DECISION MATRIX FOR 15 CRITERIA

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	[0.7000,...	[0.800...	[0,0,0...	[0.4000,...	[0.500...	[0,0.5...	[0.1000,...	[0.50...	[0,0,0....	[0,0,0....	[0.40...	[0,0,0.1...	[0,0,0.5...	[0,0,0.1...	[0.400...
2	[0.7000,...	[0.700...	[0.10...	[0.7000,...	[0.200...	[0.600...	[0.2000,...	[0,0,0...	[0.300...	[0.200...	[0,0,0...	[0,0,0.3...	[0,0,0.5...	[0.2000,...	[0.500...
3	[0,0,0.10...	[0.800...	[0.30...	[0.8000,...	[0.700...	[0.800...	[0.7000,...	[0.80...	[0.700...	[0.800...	[0,0,0...	[0.5000...	[0,0,0.6...	[0,0,0.8...	[0.500...

TABLE 10. NONSOCIAL WEIGHTED MATRIX, THE LARGEST AND SMALLEST NUMBER OF EACH COLUMN FOR 15 CRITERIA

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	[0,0,0.3773]	[0,0.57...	[0,0,0....	[0.1899...	[0,0,0.4...	[0,0.15...	[0.0631,...	[0,0,0.1...	[0,0.03...	[0,0.0...	[0,0,0....	[0,0.0...	[0,0,0...	[0,0,0.0...	[0,0,0....
2	[0,0,0.4178]	[0,0.51...	[0,0,0....	[0.3323...	[0,0,0.1...	[0.160...	[0.1262,...	[0,0,0.0...	[0.083...	[0.102...	[0,0,0....	[0,0.2...	[0,0,0...	[0,0,0.1...	[0,0,0....
3	[0,0,0.0270]	[0,0.58...	[0,0,0....	[0.3798...	[0,0,0.5...	[0.213...	[0.4418,...	[0,0,0.2...	[0.194...	[0.410...	[0,0,0....	[0.41...	[0,0,0...	[0,0,0.2...	[0,0,0....
max	[0,0,0.4178]	[0,0.58...	[0,0,0....	[0.3798...	[0,0,0.5...	[0.213...	[0.4418,...	[0,0,0.2...	[0.194...	[0.410...	[0,0,0....	[0.41...	[0,0,0...	[0,0,0.2...	[0,0,0....
min	[0,0,0.0270]	[0,0.51...	[0,0,0....	[0.1899...	[0,0,0.1...	[0,0.15...	[0.0631,...	[0,0,0.0...	[0,0.03...	[0,0.0...	[0,0,0....	[0,0.0...	[0,0,0...	[0,0,0.0...	[0,0,0....

TABLE 11. RANKING DIFFERENT CHOICES FOR 15 CRITERIA

	1	2	3	4
1	2.8317	0.4239	0.1302	3
2	1.9399	1.1689	0.3760	2
3	0.3919	2.8537	0.8793	1

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