

A new approach for data visualization problem

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Abstract — Data visualization is the process of transforming data, information, and knowledge into visual form, making use of humans' natural visual capabilities which reveals relationships in data sets that are not evident from the raw data, by using mathematical techniques to reduce the number of dimensions in the data set while preserving the relevant inherent properties. In this paper, we formulated data visualization as a Quadric Assignment Problem (QAP), and then presented an Artificial Bee Colony (ABC) to solve the resulted discrete optimization problem. The idea behind this approach is to provide mechanisms based on ABC to overcome trapped in local minima and improving the resulted solutions. To demonstrate the application of ABC on discrete optimization in data visualization, we used a database of electricity load and compared the results to other popular methods such as SOM, MDS and Sammon's map. The results show that QAP-ABC has high performance with compared others.

Keywords - Data visualization, Quadric Assignment Problem (QAP), Artificial Bee Colony (ABC)

I. INTRODUCTION

Data visualization is a graphical presentation of multidimensional data making use of humans' natural visual capabilities. More specifically, data visualization reveals relationships in data sets that are not evident from the raw data, by using mathematical techniques to reduce the number of dimensions in the data set while preserving the relevant inherent properties. Compared with the data appear in their original high dimensional space, we may be more capable of finding the possible relationship among the data points in the low dimensional space, i.e., 2-D. therefore, it has been widely applied to solve many problems, e.g. signal compression, pattern recognition, image processing, etc[1].

Dimensionality reduction or projection techniques can transform large data of multiple dimensions into a smaller, more manageable set with special methods. Principal component analysis (PCA) and MultiDimensional Scaling (MDS) are two popular methods for data reduction and visualization. PCA is one of the most widely used methods because it is effective and robust for performing linear projection. Linear projection means the projection of data is conducted by multiplying each component of the original vector with a scalar. Thus, PCA is not the most suitable approach when one is dealing with highly nonlinear data. MDS is another classical projection but its final visualization map is difficult to perceive, when one is handling high dimensional and highly unsymmetrical data set. Sammon's mapping is the earliest approach in nonlinear projecting data into low dimensional space for visualizing multivariate data. Sammon's mapping tries to minimize the

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distances between input data in the original high dimensional space and the output data in the projected space. It is capable of preserving the topological structure of the original data and their corresponding inter point distances, but Sammon's mapping is computationally demanding especially when one is handling huge numbers of data points. In addition, it requires re-computation when new data points are added [2, 3].

Discrete optimization techniques provide a possible alternative for the data visualization problem. Discretizing the data visualization problem may result in a large-scale quadratic assignment problem that is very difficult to solve to optimality [4]. The complexities of these problems are high and it is a NP Complete. One approach to overcome high large-scale quadratic assignment and its complexity is solving it by heuristics or intelligent optimization. As more and more real-world optimization problems become increasingly complex, algorithms with more capable optimizations are also increasing in demand. We formulate the data visualization problem as a Quadratic Assignment Problem (QAP) and then use the Artificial Bee Colony (ABC) to solve it. The idea behind this approach is to provide mechanisms based on ABC to overcome trapped in local minima and improving the resulted solutions. To evaluate our method we use a real data set and compare our proposed hybrid approach which is named QAP-ABC with SOM, MDS and Sammon's Map as popular methods for data visualization.

The rest of the paper is organized as follows. Section 2 reviews relevant works in data visualization. Section 3 introduces modeling the problem as a Quadratic Assignment Problem and discusses Artificial Bee Colony (ABC) to solve this problem in detail. Section 4 presents a case study to evaluate the technique with experimental results, and then compares it with Self-Organizing Maps (SOM), Sammon's Map (SM) and MultiDimensional Scaling (MDS) in section 5. Section 6 concludes the paper.

II. Background

The most common data visualization methods allocate a representation for each data point in a lower-dimensional space and try to optimize these representations so that the distances between them are as similar as possible to the original distances of the corresponding data items. The

methods differ in that how the different distances are weighted and how the representations are optimized. Linear mapping, like principle component analysis, is effective but cannot truly reflect the data structure. Non-linear mapping, like Sammon's Mapping (SM), MultiDimensional Scaling (MDS) and Self-Organizing Map (SOM) [5, 6], requires more computation but is better at preserving the data structure.

1. Sammon's Mapping (SM)

Sammon Jr. [7] introduced a method for nonlinear mapping of multidimensional data into a two- or three-dimensional space. This nonlinear mapping is a MultiDimensional Scaling (MDS) projection which preserves approximately the inherent structure of the data and thus is widely used in pattern recognition. It tries to match the pairwise distances in the lower-dimensional representations with their original distances [3].

Denote the data vector $x_i = (x_{i1}, x_{i2}, \dots, x_{id})^T$, $i = 1, \dots, N$ (N is the number of data points) in a d -dimensional input space and the corresponding vector $y_i = (y_{i1}, y_{i2}, \dots, y_{id})^T$ in a 1-dimensional output space ($l=2$ or 3). Define the distance between x_i and x_j in the input space as $d_{ij}^* \equiv d(x_i, x_j)$ and the distance between y_i and y_j in the output space as $d_{ij} \equiv d(y_i, y_j)$. Distances are measured by

Euclidian metric. Error E is defined as follows:

$$E = \frac{1}{\sum_{i < j}^N d_{ij}^*} \sum_{i < j}^N \frac{(d_{ij}^* - d_{ij})^2}{d_{ij}^*} \quad (1)$$

Error E is minimized such that the inter-point distances in the output space approximate the corresponding distances in the input space. After mapping, the inherent structure of input data is preserved and visualized in the output space. But Sammon's nonlinear projection is unable to provide an explicit mapping function, making the projection of new added data require re-computation. Besides, the distance calculations of $N(N-1)/2$ times result in that the computational complexity $O(N^2)$ is large. Even with today's

fast computers, nonlinear optimization techniques are usually slow and inefficient for large data sets. In addition, since the error is minimized by the steepest descent procedure, it is easily trapped in local minima [2, 3, 8].

2. MultiDimensional Scaling (MDS)

Multidimensional Scaling (MDS) refers to a class of algorithms that visualize proximity relations of objects by distances between points in a low-dimensional Euclidean space. MDS algorithms are commonly used to visualize proximity data (i.e., pairwise dissimilarity values instead of feature vectors) by a set of representation points in a suitable embedding space [9, 10].

Let $D = \{x_1, x_2, \dots, x_n\}$ be a set of n objects in a d -dimensional feature space, and let d_{is} denote the Euclidean distance between x_i and x_s . Let d^* be the number of dimensions of the output space, x_i^* be a d^* -dimensional point representing x_i in the output space, and d_{is}^* denote the Euclidean distance between x_i^* and x_s^* in the output space. Let X be an $n \times d$ -dimensional vector denoting the coordinates x_{ij} ($1 \leq i \leq n, 1 \leq j \leq d$) as follows:

$$x_i = (x_{i1}, x_{i2}, \dots, x_{id})^T, \tag{2}$$

Where $x_{(i-1)d+j} = x_{ij}$ for $i=1,2,\dots,n$ and $j=1,2,\dots,d$
 Error E_{MDS} is defined as follows:

$$E_{MDS} = \frac{\sum_{1 \leq i \leq s \leq n} (d_{is}^* - d_{is})^2}{\sum_{1 \leq i \leq s \leq n} d_{is}^2} \tag{3}$$

The only difference between Sammon's Mapping and the nonlinear metric MDS is that (excluding the constant normalizing factor) the errors in distance preservation are normalized by the distance in the original space. Due to normalization, the preservation of small distances will be emphasized [9, 11].

The minimization of Error E_{MDS} is a optimization problem which is non-convex and sensitive to local minima and the main limitation of most MDS applications is that it requires $O(N^2)$ memory as well as $O(N^2)$ computation [12]. Thus, though it is possible to run them with small data size without any trouble, it is impossible to execute them with a large number of data due to memory limitation; therefore, this challenge could be considered as being a memory-bound problem. Moreover, the MDS final visualization

map is difficult to perceive, when one is handling high-dimensional and highly unsymmetrical data set [1, 13].

3. Self-Organizing Maps (SOM)

SOM, proposed by Kohonen [14], is a class of neural networks trained in an unsupervised manner, using competitive learning. It is a well-known method for mapping a high-dimensional space onto a low-dimensional one. It is popular to consider mapping onto a two-dimensional grid of neurons. The method allows putting complex data into order, based on their similarity, and shows a map by which the features of the data can be identified and evaluated. A variety of realizations of SOM has been developed [5, 6].

The SOM architecture consists of two fully connected layers: an input layer and a Kohonen layer. Neurons in the Kohonen layer are arranged in a one- or two-dimensional lattice. Fig. 1 displays the layout of a one-dimensional map where the output neurons are arranged in a one-dimensional lattice. The number of neurons in the input layer matches the number of attributes of the objects. In the input layer each neuron has a feed-forward connection to each neuron in the Kohonen layer [3].

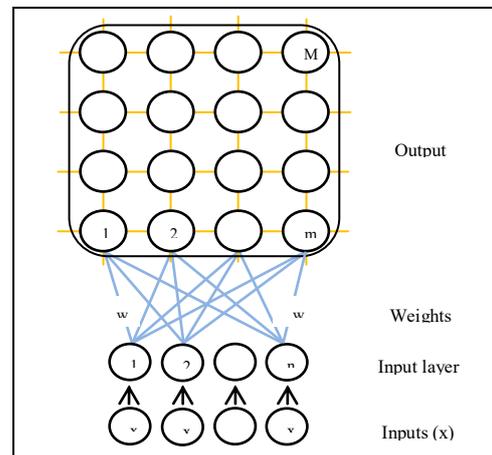


Fig. 1. The structure of Self-Organization Map (SOM)

Kohonen's SOM and Sammon's nonlinear mapping are topology- and distance-preserving mapping techniques commonly used for multivariate data projections. However, the computations for both techniques are high.

III. Data Visualization Problem: Definition and Formulation

Discrete optimization techniques provide a possible alternative for the data visualization problem. Discretizing the data visualization problem may result in a large-scale Quadratic Assignment Problem that is very difficult to solve. Roselyn Abbiw-Jackson [4] used the divide and conquer algorithm to solve the Quadratic Assignment Problem (QAP). We concentrate solve data visualization problem as discrete optimization.

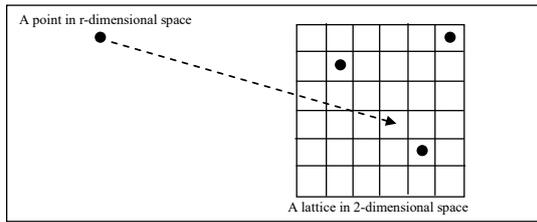


Fig. 2. Structure of discretizing the data visualization problem

Let M be a set with n points in r -dimensional space (R^r). The data visualization problem is locating any r -dimensional point in M to q -dimensional space (R^q , $q < r$ and $q=2$ or 3) such that a relevant measure of distance is preserved (Fig. 2). We use discrete optimization techniques to solve it. Therefore, we approximate the continuous q -dimensional space by a lattice N while each cell has a center point. On the other hand, the data visualization problem is similar to assigning m point to n cell (center) points. The decision variables are given by:

$$x_{ik} = \begin{cases} 1, & \text{if the } i^{\text{th}} \text{ instance is assigned to lattice point } k \in N \\ 0, & \text{Otherwise} \end{cases} \quad (4)$$

Therefore, data visualization problem can be written as follows:

$$\min \sum_{i=1}^m \sum_{j=1}^m \sum_{k=1}^n \sum_{l=1}^n F(D_{ij}^{old}, D_{ij}^{new}) x_{ik} x_{jl} \quad (5)$$

subject to $\sum_{k \in N} x_{ik} = 1, \forall i \in M$

$$x_{ik} \in \{0,1\},$$

Where, $D^{old} \in R^m \times R^r$ is a matrix

measuring the distance between n given instances, $D^{new} \in R^n \times R^q$ is a new distance matrix

between assigned instances in the $q=2$ or 3 dimensional space and F is a function of the deviation between the differences between the instances in the original space and the new q -dimensional space. Choices for F include the functions for Sammon's Mapping and classical scaling, and all objective functions for non-metric scaling. By using Sammon's Mapping as objective function, data visualization problem can be formulated as:

$$\begin{aligned} \text{Min } & \frac{1}{\sum_{i=1}^m \sum_{j=1}^n d(i,j)} \sum_{i=1}^m \sum_{j=1}^n \sum_{k=1}^n \sum_{l=1}^n \left(\frac{(d(i,j) - d^*(k,l))^2}{d(i,j)} \right) x_{ik} x_{jl} \\ \text{subject to } & \sum_{k=1}^n x_{ik} = 1, \forall i \in M \\ & x_{ik} \in \{0,1\}, \end{aligned} \quad (6)$$

Where, $d(i,j)$ is distance (usually Euclidean distance) between original points in space of R^r and $d^*(k,l)$ is distance between lattice points in space of R^q .

The number of cells is determined with regards to requirement of data visualization method. A problem with points spread out will require a larger grid than one with points clustered together. The larger the grid, the more accurate the final result. To scale the cells (in q -dimensional space) and the given data set, we find the greatest distance between the pairs of points in given data set. Let this distance be a and the greatest distance in the chosen lattice N be b . Then, we multiply all original distances between points in M by b/a , so that our lattice is scaled to the given problem. The problem of assigning m points to n lattice points cannot be treated as a linear assignment problem because a linear assignment problem assumes that the cost of assignment of one point to a lattice point does not depend on the assignment of the other points. However, this is not the case for data visualization problems. Therefore, this problem is a Quadric Assignment Problem (QAP) whose objective function is convex and it can have many local solutions. On the other hand, space of this

problem is very large and decision variables are internally depended.

Any solution method can be used to solve this problem but it should be effective in circumstance of problem. Proposed solution method here is Artificial Bee Colony named QAP-ABC for solving data visualization problem as a discrete optimization problem. The idea behind this approach is to provide mechanisms for improving the data visualization precise. To introduce the proposed method, it is necessary to give a brief review of Artificial Bee Colony and its framework.

III. Proposed Method: QAP-ABC

The Artificial Bee Colony (ABC) method is inspired by the intelligent foraging behavior of a honey bee colony in which bees try to maximize the nectar amount loaded into the hive by bees assigned a specific task. In the foraging behavior, there are three types of bees, each of which has a different search characteristic: these are employed bees, onlooker bees and scout bees. The employed bees are responsible for exploiting the sources in their memory whereas the onlooker bees go to exploit potentially rich sources depending on the information taken by communicating with the employed bees. Scout bees are responsible for exploring undiscovered sources by a random motivation or external clue [15, 16].

The ABC algorithm mimics this foraging behavior of honey bees in the context of a global optimization algorithm. From the perspective of a meta-heuristic, a population of solutions corresponds to the food sources to be discovered and the optimization task is to find the most profitable source by using the different search characteristics of the employed bees, onlooker bees and scout bees [17-19]. Each type of bee is a phase of the algorithm as given in Fig. 3.

The perturbation strategy in ABC approach by using of calculation of x'_{ij} provides the solution

to move towards promising regions of the search space. This is the positive feedback of the ABC algorithm. By a greedy selection in step 7, the new solution is kept in the memory and the current one is discarded if the new solution is better; otherwise, it is discarded, and the current one is retained in the population and a counter associated with the current solution is incremented

by one in order to count the number of nonprogressive local searches in the neighborhood of the current solution. This counter will be used to determine the exhausted sources to be used in the phase for scout bees. In the phase for employed bees, the local searches are conducted for all solutions in turn, while in the phase for onlooker bees they are carried out for the solutions chosen probabilistically. This probabilistic selection provokes high quality solutions to be chosen more, but also allows poor solutions to be selected. This selection scheme is also another positive feedback of the ABC algorithm [17, 20]. It is clear from the above explanation that there are three control parameters in the basic ABC: The number of food sources which is equal to the number of employed or onlooker bees (SN), the value of dimension of the problem (D) and the maximum cycle number (MCN).

In our proposed approach (QAP-ABC), we transform the data visualization problem to a Quadratic Assignment Problem (QAP) and apply ABC algorithm to optimize it. Then use the best bee in final phase of ABC algorithm for projection of original data points to a map and providing the output lattice. The Block Diagram of QAP-ABC is presented in Fig. 4.

As it is seen in Fig. 4, our proposed approach has three phases which detailed explained in follows:

1. Preprocessing

Since it is possible that the used data set has been comprised missed, inconsistency or noisy data, thus it is required to improve the quality of the actual data for mining by data pre processing. This also increases the mining efficiency by reducing the time required for mining the preprocessed data. Data preprocessing involves data cleaning, data integration, data transformation, data reduction, data normalization and etc. this phase is done same approach in [21].

2. Projection of Data Points in r-Dimensional Space to q-Dimensional Space

After preprocessing data phase, high dimensional data points in r-dimensions, the number r of attributes is large, projected to low dimensional (q-dimensional) space whereas the data can only be visualized in two or three dimensions. This reduction of dimensions should do in a way that preserves the structure of the relationships (that is, distances) between original

Before introducing the ABC algorithm, it is necessary to give used notation in this algorithm. Let,

k: A uniform random index from $[1, CS]$ interval and it has to different from i .

CS: The number of food sources.

j: The uniform random index that is belong to $[1, D]$.

D: The dimension of the problem.

fitness_{*i*}: The fitness value of the solution i .

1: Initialize the population of solution x_{ij}

$$x_{ij} = x_j^{min} + \text{rand}(0, 1)(x_j^{max} - x_j^{min}) \tag{7}$$

2: Evaluate the population

3: Cycle = 1

4: Repeat

5: Produce new solution x'_{ij} for the employed bees using following equation:

$$x'_{ij} = x_{ij} + \phi_{ij} (x_{ij} - x_{kj}) \tag{8}$$

6: Evaluate the population

7: Apply the greedy selection process between x_{ij} and x'_{ij}

8: Calculate the probability values p_i as follows:

$$p_i = \frac{\text{fitness}_i}{\sum_{i=1}^{CS} \text{fitness}_i} \tag{9}$$

9: Produce new solution x'_{ij} for the onlookers depending on p_i

10: Evaluate the population

11: Apply the greedy selection process between x_{ij} and x'_{ij}

12: Determine the abandoned solution, if exists, and replace it with a new randomly produced solution x_i for the scout.

13: Memorize the best food source position achieved so far

14: Cycle = Cycle + 1

15: Until Cycle = Maximum Cycle Number or time = Maximum CPU time

Fig. 3. Pseudo code of the basic ABC algorithm

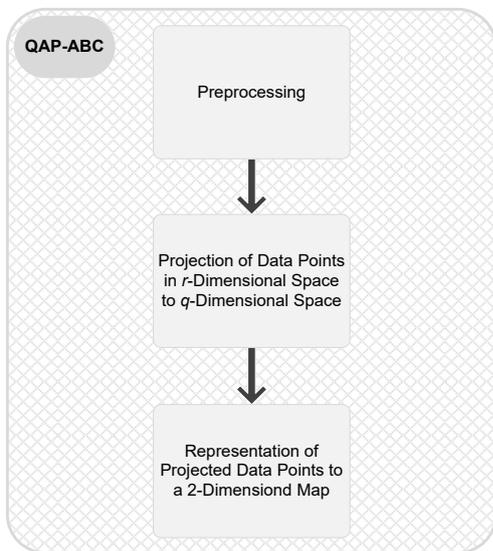


Fig. 4. Block Diagram of QAP-ABC

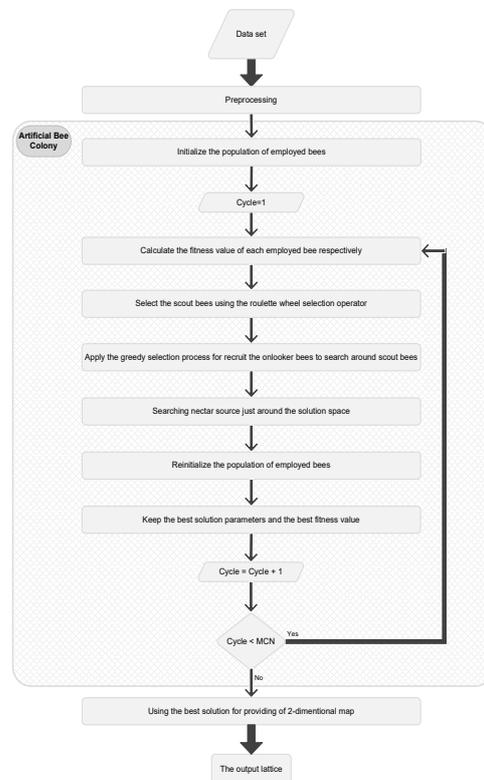


Fig. 5. Flow chart of QAP-ABC

data points. We concentrate to this phase in our article. The flowchart of our proposed approach to solve data visualization problem using of ABC algorithm is presented in Fig. 5.

3. Representation of Projected Data points to a 2-Dimension Map

We use the best bee resulted from ABC to project the high dimensional data points in r -dimensions to 2-dimensionals ($q=2$) space. As it is presented in Fig. 6, the best bee has $366 \times 2 = 732$ cells that i th pair of cells corresponds to the coordinate of i th original data points in output map which $i=1,2,3,\dots,366$. The best resulted solution from ABC has minimum error, thus it guaranty to preserving the structure of the relationships between original data points in output map.

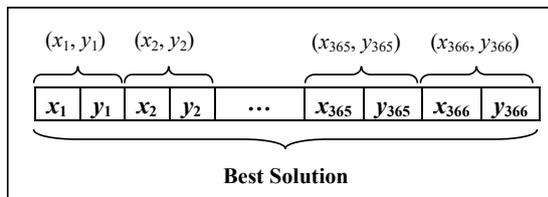


Fig. 6. The best solution resulted from ABC representation

IV. Experimental Results

This section evaluates the performance of the QAP-ABC, SOM, MDS and Sammon's Map to data visualization. Experiments were performed using an Intel Core 2 Duo, 2.1 GHz processor with 2 GB of RAM.

1. Data Set

In order to compare the proposed technique (QAP-ABC) and the well-known methods such as Self-Organizing Maps algorithm (SOM), Sammon's Map and MDS, a real-life case is considered here. Used data set in this research includes 24-hour electrical load of 366 days starting from 20 March 2008 in Esfahan. Therefore, we have 366 instances with 24 attributes, which any attribute is as an hour (a label number for all the daily load curves).

2. Parameters Setting

The dimension of the output map in this research is assumed to be two ($q=2$), because most visible media (for example: paper, monitor panel and etc.) are 2-dimensional. The output

map is a 60×60 grid square. Dimension of the output map is calculated by a heuristic function in SOM toolbox in MATLAB [22].

ABC settings: Except common parameter (maximum cycle number), the basic ABC used in this study employs two control parameters D and SN . The maximum cycle number is supposed 2000 and other ABC parameters is resulted from [18] as: In original ABC algorithm, 50% of the colony consists of employed and the rest 50% consists of onlooker bees. In other words, the ratio of employed and onlooker bees are the same, 1:1 [23]. Therefore, we use ratio 1:1 in this study. To reach the optimal parameters we run ABC with different parameters D and SN . The ABC program was run 30 times and average of observations register for any parameters. Table 1 and Fig. 7 show the results of original (ratio 1:1) ABC algorithms with different D and SN .

Table 1. Fitness values for QAP-ABC with $SN=25$ and various number of dimensions

number of dimensions (D)	Fitness
2	0.0613
5	0.0543
10	0.0513
20	0.0489
30	0.0496
50	0.0506
100	0.0519

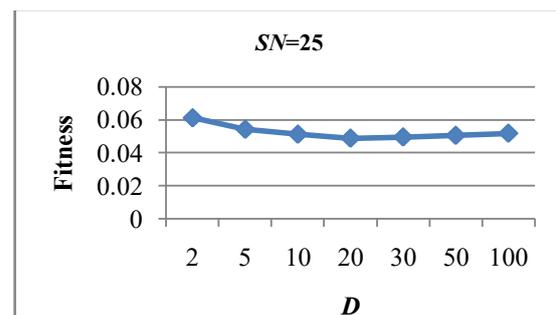


Fig. 7. The sensitivity of ABC to number of dimensions (D) in QAP-ABC

The result of QAP-ABC with $SN=25$ and various number of dimensions (D) is shown in Fig. 7 and Table 1. As it is shown in Fig. 7 and Table 1 more number of dimensions improves the fitness. The decreasing of fitness by increasing the number of dimensions continues until D reach to 20. Therefore we propose $D=20$ for ABC. To determine the SN , we run ABC program with

$D=20$ and different SN (see Table 2 and Fig. 8).

Table2. Fitness values for QAP-ABC with $D=20$ and various number of onlooker bees

Onlooker Bees (SN)	Employed bees	Fitness
10	10	0.0541
25	25	0.0489
50	50	0.0439
75	75	0.0395
100	100	0.0351
125	125	0.0332
150	150	0.0327
175	175	0.0327
200	200	0.0327

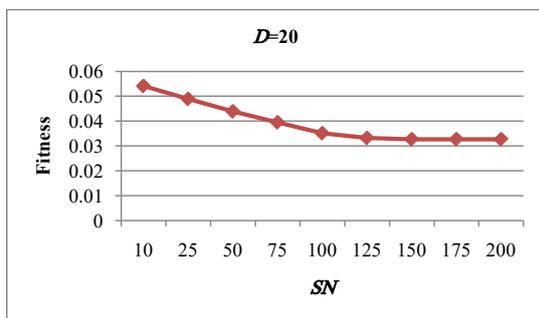


Fig. 8. The sensitivity of ABC to number of onlooker bees (SN) in QAP-ABC

As it is shown in Table 2 and Fig. 8, increasing the number of onlooker bees (SN) improve the fitness of resulted solutions. The fitness is unchanged for number of onlooker bees greater than 125. Thus we propose $SN=125$ in our study.

SOM settings: The SOMs were trained using the batch training algorithm in two phases: (1) rough training phase which lasted 1000 iterations with an initial neighborhood radius equal to 5, a final neighborhood radius equal to 2, While a learning rate starts at 0.5 and end at 0.1, and (2) fine training phase which lasted 500 iteration cycles, While a learning rate starts at 0.1 and end at 0.02, we do set the neighborhood partitions started at 2 and end with 0.

V. Quality of Data Visualization

To illustrate the effectiveness of the proposed method (QAP-ABC), the precise results of used data visualization methods for several data sets that are randomly selected from original data set are shown from **Error! Reference source not found.** Number size of data sets is 50-100% (increment 10) of total records number. All of methods run for 1500 seconds. As it is

seen in Table 3, Sammon's Map couldn't reach to solution in limited iteration (2000 iterations) for large data bases (above 70%) and defect in convergence. Moreover, it is similar results for MDS. The MDS is defected for above 60% to reach solution in regarded iteration.

As shown in Error! Reference source not found., Sammon's Map and MDS although have a high precise but those couldn't be success for large data set and don't convergent and give "Iteration limit exceeded. Minimization of criterion did not converge" error after 2000 iterations. Because, used nonlinear optimization techniques for MDS and Sammon's Map are usually inefficient for large data sets. In addition, since nonlinear optimization techniques need high computational time, MDS and Sammon's Map are slow.

Fitness value, which calculates the coefficient error of preserving of the approximately inherent structure of the data by dimension reduction, is 0.0327 and 0.0563 for QAP-ABC and SOM, respectively (Fig. 9). It can be clearly observed that the QAP-ABC has been better than SOM and other used methods and 41% improvement in the fitness values over SOM.

In ABC, The onlooker ants select scout bees based on elite selection. The elite selection preserves the best solutions without any change in next step. The transforming of the best solutions to next step and the ignorance of the worst solutions cause to search increase for bees in the high fitness region of the state space, increases thus improving exploitation. Moreover, number of soldier bees is determined based on the fitness of scout bees. Thus the best scout bees have more soldier bees for search. It result concentrate to better promising region of search space (exploitation) and decrease the computational time and increase the convergence speed. This is because the QAP-ABC increases the exploitation from information of meet points by using of positive feedback. It should also be noted that QAP-ABC has higher computational time compared with SOM.

VI. CONCLUSION

In this work we suggested a new approach based on Artificial Bee Colony and Quadric Assignment Problem named QAP-ABC for data visualization problem. Proposed method with other popular data visualization method, SOM, MDS and Sammon's Map, are executed on a real

data set with same circumstances. The results were compared and it showed that QAP-ABC has high performance with compared others.

Table 3. The data visualization precise of various approaches

Dataset size	QAP-ABC		SOM		Sammon's Map		MDS	
	Fitness	Running Time	Fitness	Running Time	Fitness	Running Time	Fitness	Running Time
50	0.0261	3215	0.0354	732	0.0262	12040	0.0264	11510
60	0.0263	4587	0.0373	762	0.0265	12350	-	-
70	0.0275	6051	0.0395	775	-	-	-	-
80	0.0285	7562	0.0435	782	-	-	-	-
90	0.031	8541	0.489	793	-	-	-	-
100	0.0327	10578	0.0563	1015	-	-	-	-

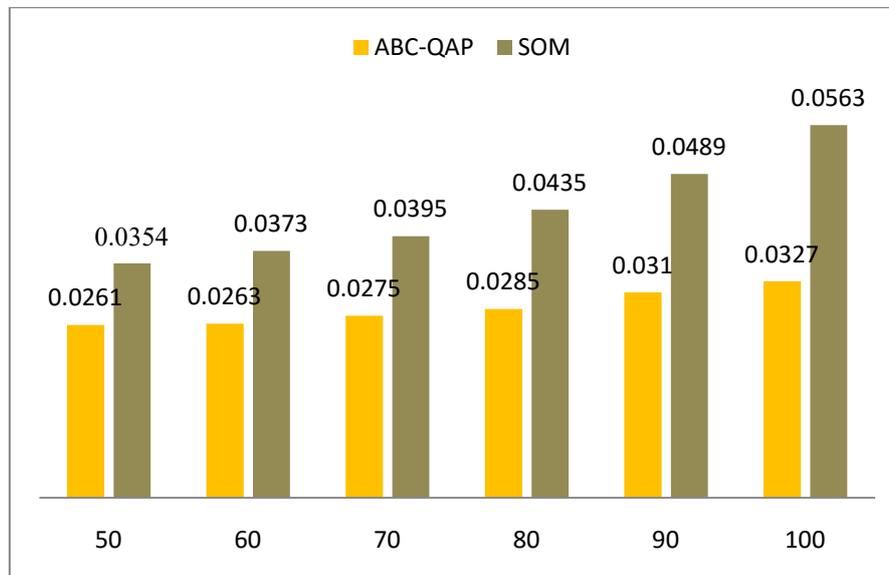


Fig. 9. To compare the results of SOM and QAP-ABC

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